Preference Aggregation Based Cognitive Modeling

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Abstract

This paper proposes a cognitive modeling based on preference aggregation, which is applied to the Wason selection task (WST) a well-known experiment on human reasoning about conditionals. The notion of a cognitive rights system, which distributes the legitimacy, or authority, of inspection on each card by majority rule, is introduced. The normative solution can be predicted as the no-winner result, i.e., a Condorcet cycle. It can thereby be explained that the poor performance and the production of cognitive biases in WST are caused by malleability of the ordering. Further, the permission schema facilitates normative selection, because it is proof against malleability.

Keywords: conditional; human reasoning; Wason selection task; majority vote; cognitive rights system

1. Introduction

Conditionals are building blocks of logical, or hypothetical, human reasoning (Holland et al, 1986; Evans, 1989; Evans and Over, 2004). They are also important to the model of decision making because, in normative theory at least, it is commonly considered that transitivity is the most important property. Much psychological evidence has refuted it in both fields during the last century. Nonetheless transitivity would be significant in cognitive modeling regarding our self-knowledge in action, i.e., "I know what I know" and "I don't know what I don't know."

In other words, self-knowledge means "My knowledge representation cannot manipulate myself about the information contents I have."

This is often not the case in ordinary thought. Peter Wason invented several celebrated experiments on human reasoning, especially, the Wason Selection Task, or the four-card selection task (abbreviated to WST) on hypothetical testing (Wason, 1966).

A	B	7	8

The Rule: "If there is an A on one side of the card, then there is a 7 on the other side of the card." Choose those cards that need to be turned over to decide whether the rule is true or false.

Figure 1: Wason Selection Task.

The puzzle of WST demonstrates firstly that it would be very difficult to check an affirmative indicative conditional being out of touch with reality. In Figure 1, the four cards are shown with their exposed sides. According to the literature, even though most subjects agree with the experimenter that an "A" on one side and an "8" on other side (in schematic form, "*p*" and "not *q*") is a violation, the correct solution rate observed is typically between 5% and 20%. Frequently observed patterns are "only A" and "both A and 7" (in schematic form, "*p*" and "*p* & *q*").

This may be nothing surprising in that human beings are prone to error. However, experimental studies on different versions of WST reported by psychologists have drawn our attention to the fact that the performance in WST is remarkably improved by introducing negations, domain-dependent, thematic, or deontic contents (Evans and Over (2004) Chapter 5; Holland et al. (1986) Chapter 9). [1]

In this paper, a new cognitive modeling based on preference aggregation, which has been studied for social choice and voting procedures (Arrow, 1963; Sen, 1982; Gaertner, 2001; Taylor, 2005), is applied to human reasoning, especially to WST. I also developed a computational implementation, making use of PROLOG, a Horn clause logic programming language (Clocksin and Mellish, 2003).

In the next section I introduce two notions, the pre-diagnostic process and the choice procedure, by means of which WST can be seen as a decision making problem. Section 3 recasts it as a preference aggregation by using a pairwise majority vote and solves WST by the Condorcet cycle. Section 4 introduces another important notion, the cognitive rights system, which interprets the aggregation as distributing authority of inspection for each card. In Sections 5 and 6, both the normative solution and the cognitive biases for WST are simulated with my PROLOG code. In Section 7 the stability of the permission schema is interpreted. Lastly Section 8 concludes the paper.

2. Conditionals and Pre-diagnostic Process

In logics, and in mathematics, an indicative conditional represented as the sentence 'If *p* then *q*' should be read as a material conditional, $p \rightarrow q$[2], i.e., 'If the antecedent *p* is true, then the consequence *q* is true.' A material conditional has its truth function f defined as follows: f:B×B→B, s.t., $f(p\&q)= f((\text{not } p)\&q)=f((\text{not } p)\&(\text{not } q))=\text{T}$, and $f(p\&(\text{not } q))=\text{F}$, where B={T,F}, and T, F represent truth, and falsity respectively.

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[1] These studies lead to a number of cognitive modelings on conditional reasoning, for example, the Mental Model Theory (Johnson-Laird and Byrne, 2002), the Pragmatic Reasoning Schema (Cheng and Holyoak, 1985), the Relevance Theory (Sperber et al., 1995), the Darwinian Algorithms (Fiddick et al., 2000), and the Dual Process Theory (Evans and Over, 2004).

[2] Synonymously '*p* only if *q*,' or 'Every *p* is *q*.'

As noted in the preceding section, the normative solution for WST assumes that the indicative conditional in Figure 1 has the truth function of a material conditional. And the procedure to solve WST may be seen as a scientist's hypothetical reasoning (Popper, 1959), which suppresses inspection unless there is a hope to refute it.

In our modeling, firstly, WST is translated into a two-stage decision making problem. The subject of WST can be seen as a decision maker (DM) who should decide whether or not to select for each card. I expect the reader to comprehend that it is rather obvious that one can not solve WST without some procedural knowledge other than the truth table (i.e., the semantic knowledge) of a conditional sentence.

We then assume that before each inspection, a DM of WST constructs a knowledge representation system which provides useful information for decision making. [3]

I would like to call this setup stage a pre-diagnostic process, followed by the subsequent choice procedure. Knowledge representations produced by a pre-diagnostic process are used in the choice procedure, which eases up DM on the task, and provides reason, or legitimacy, which justifies the final choice, if DM cannot fully achieve the given goal.

Let X denote a set of possible actions and D a set of possible data available to DM. The WST is to select d:D→X, a function of the data set D to the action set, where X={'to inspect', 'not to inspect'}, D={p, q, *not p*, *not q*}.

Formally, a pre-diagnostic process is defined as h:D→M, a function of D into M, where M denotes the mental model space of the subject regarding the cards. For any data $w \in$ D, we call $h(w) \in$ M a concerning set, or a mental model of the cognitive representation system that DM has generated. The choice procedure is defined as g:M→X, so $d= g \circ h$.

Throughout this paper, it is assumed that preference (or ordering) aggregation, especially majority rule, models the pre-diagnostic process h, and the succeeding choice procedure, for WST.

3. Aggregation Procedure

In Table 1, unit values and zero values represent authorized inspections and default beliefs respectively, observing a data in each column label (See Table 1). The first three rows, R1, R2, and R3 in Table 1, are components, or *reasons*, in M created by the pre-diagnostic process h for each card, and the last row is the aggregated one by majority rule, which stands for the *final* selection.

By the choice procedure g the DM chooses simply to inspect if the value is 1, which indicates the concern regarding the inspective mission, otherwise not to inspect. A 0-value, therefore, means an 'is-believable' relation.

Note that a p is equivalent to T→p and a *not q* is equivalent to q→F(→T). Each column in Table 1 represents the binary majority vote regarding the ordering of two basic propositions. R1 in the first row of Table 1 reproduces confirmation bias, or matching bias (in an affirmative statement), which has been observed in the literature. R3, which is opposite to this bias, is selects two mismatch cases (not p and *not q*). The middle row, R2, is the case memory which violates the rule, however, biased with the confirmation.

Table 1: An aggregation procedure for WST.

Reason	p	q	$p \to q$	$\neg p$	$\neg q$	$q \to p$
R1	1	1	1	0	0	0
R2	1	0	0	0	1	1
R3	0	0	1	1	1	0
Majority (M)	1	0	1	0	1	0

Clearly it is shown in Table 1 that the normative solution for WST can be implemented under a majority decision. And the majority decision has no winner, i.e., a Condorcet cycle (Sen, 1982; Gaertner, 2001). [4] Thus the normative reasoning of a scientist to refute $p \to q$ can be implemented by a cyclic majority vote by micro-agents who have simple, near-sighted motivations.

Further, we can explain the poor performance of ordinary people in WST, assuming they can represent justifying reasons as unrestricted orderings over propositions. In fact, the probability of a cycle is about 5.6% for 3-person and 3-candidate cases and less than 8.8% for any 3-candidate cases (See Gaertner (2001) p.37, Table 3.1).

4. Cognitive Rights System

In our modeling, the pre-diagnostic process generates a knowledge representation system to distribute the authority of inspection (1-values), and the protected privacies against inspection (0-values), in other words, believable cards, respectively in the majority row of Table 1. Note that the ordering here is not merely considered as preference (or belief); rather, the priority among rights, or relative relevance, concerning it may not harm other important rights. This provides a limited, but nimble reasoning ability for adaptive decision making in the succeeding step.

The unconscious feature of this first step seems to shed light on the relevance-creation system. A failure to create the relevant information, which represents a possible violation of the rule and awakes the subject to the risk, explains the poor performance in WST. [5]

[3] This stage can be considered as being processed, often unconsciously, in ordinary thinking. The Heuristic-Analytic Process and the Dual Process (Evans, 1989; Evans and Over, 2004) are similar to mine, but my theory does not commit to evolutionary theory or brain scientific apparatus.

[4] Table 1 may be seen as a 'doctrinal paradox' (Kornhauser, 1992), or a 'judgment aggregation' (Diederich, 2006), which is recently being intensively studied. Here I regard it not as a paradox but rather a model of the normative solution for WST.

[5] Sperber and their collaborators (Fiddick et al., 2000) define relevance as a net cognitive effect in linguistic comprehension, but does not consider the aspect of the rights system.

Further, the linguistic content is considered to affect the mission or justification for inspection. In abstract WST, affirmative statements facilitate verification (confirmation), but negative statements facilitate falsification (Evans and Over, p. 77). Interestingly, cheater detection pertains to the original selection task: "to determine whether the experimenter was lying" (See Wason, p.146). It also seems contrary to the Darwinian approaches.

5. Computational Modeling

Figure 2 shows a PROLOG implementation of the three rights systems, which are represented as Table 1. And the rights to inspect are proved by resolution. See the Appendix for the source code. These programs computationally reproduce the logical content of default belief (i.e., 0-valued propositions) for each reason in Table 1, R1, R2, and R3, respectively. For each card, a failed query can be seen as the right to inspect the card.

```
%R1
r(1):p :- r(1):q.
r(1):not_p.
r(1):not_q.
```

```
%R2
r(2):q :- r(2):p.
r(2):q.
r(2):not_p.
```

```
%R3
r(3):p :- r(3):q.
r(3):p.
r(3):q.
```

```
?- to_inspect.
1: [p][q]
2: [p][not_q]
3: [not_p][not_q]
M: [p][not_q]
```

Figure 2: PROLOG programs for the rights systems.

The PROLOG system interprets each program code as a *clause* 'HEAD:-BODY.' This means logically, a rule: 'If BODY is true, then HEAD is true.' A fact clause 'FACT.' is a logical equivalent of a rule with an empty body. Each term r(K):C, which is a fact or a head of a rule, denotes a component of the K*th* row for a rights system of a subject who solves WST in Table 1.

The system tries to prove a goal as a query the user typed after a prompt ?- by using resolution-based inference with the above clauses in the internal database. The right to inspect is modeled as a failed query. For example, the first group r(1), which means R1's rights system, succeeds in queries *p* and *q* but fails in *not p* or *not q*. This fits the first row of Table 1. With a little more coding, it can be easily proved that all the authorized inspections over the available data D={p, q, *not p*, *not q*}. The lower right panel of Figure 1 shows the set of failed queries for each rights system, r(K), K=1, 2, 3. According to the majority rule it is concluded that the DM is entitled to inspect *p* and *not q*.

6. Simulating Biases

Several cognitive biases in WST, which have been found by psychologists, can be reproduced by using the previous programs. The following r(4), a slightly modified version of r(3), which stands for R3 in Table 1, together with r(1) and r(2), simulates the confirmation bias as a result of majority decision (denoted as M(i-j-k)).

```
r(4):p :- r(4):q.
r(4):p.
r(4):not_q.
```

```
➔ 4: [q][not_p]
  M(1-2-4): [p][q]
```

Alternatively, simply by discarding r(3), which suppresses the matching bias, only *p* is to be inspected.

The following r(5), a slightly modified version of r(2), which stands for R2 in Table 1, simulates the same selection of r(2) together with r(1) and r(3).[6]

```
r(5):q :- r(5):p.
r(5):p.
r(5):q.
```

```
➔ 5&M(1-5-3): [not_p][not_q]
```

For r(1), or R1, we can reverse p→q without affecting the decision.

```
r(6):q :- r(6):p.
r(6):not_p.
r(6):not_q.
```

```
➔ 6: [p][q]
  M(6-2-3): [not(q)][p]
```

This completes the six linear orderings. The preceding two models are needed to explain why introducing a negation in consequence, *p*→not *q*, facilitates falsification, but then again confirmation might occur.

Lastly, as mentioned in the introductory section, it has been reported that most subjects to select *p*, or *p* and *q*. But it is impossible to produce *p* as the single winner by a majority, except for a pair of R1 and R2, and its variants. Therefore, the pair may be considered to be the axis of bias in WST.

[6] R4 is just reversing the values both for *q* and for not *q* in R3, and R5 is just reversing the values for not *p* and for *p* in R2, respectively, in Table 1.

7. Cognitive Stability

A few more significant findings can be derived. Before expounding them, I will show the reader another graphical view of the orderings and then introduce two notions, malleability and stability.

Each double circle in Figure 3 represents his/her right of top rank priority, i.e., sovereignty. And each bold arrow represents that the order is malleable.

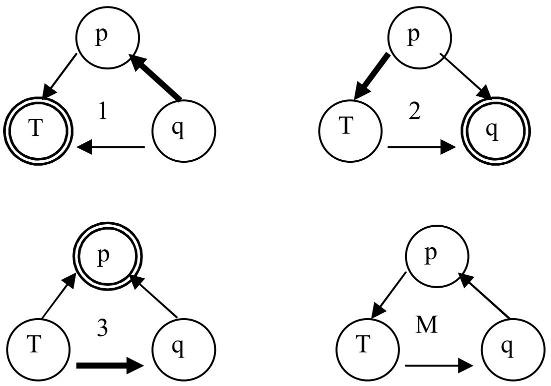

Figure 3: A graphical view of the rights system in Table 1.

In Figure 3, three bold arrows can be reversed locally without changing their top-level rights (doubly circled), respectively. That is, the bold arrows, and similarly the majority results, can be said to be *malleable*[7] with respect to the cognitive rights system. A cognitive rights system that has no malleable relations is said to be *stable*.

Malleability analysis provides us with two important observations as pieces of the puzzle in WST. Firstly, it is worth noting that only three ordering profiles suffice to reproduce the typical selection patterns, which are 1-2(*p*), 1-2-3(*p* & *not q*), and 1-2-4(*p* & *q*). [8]

Secondly, we turn our attention to the *permission schema* found by Chen and Holyoak (1985), which is a thematic content with a simple rationale to justify selection. [9] Passengers at an airport were required to show a form with a list of diseases, and it was necessary to check whether the following rule was violated. "If the form says 'ENTERING' then 'cholera' is included in the list." Suggesting the form listed inoculations, the rationale "to protect the passengers against the disease" is added.

The cognitive rights system and the supporting beliefs are as follows:

(1) The individual right of ENTERING precedes the inspecting authority to check the list of diseases. Therefore, let $(p \rightarrow q)=1$ and $(q \rightarrow p)=0$.

(2) Regarding the security mission, the authority of quarantine in order to protect the passengers precedes the individual right to ENTERING. Therefore, (not p)=$(p \rightarrow T)=0$ and $p=(T \rightarrow p)=1$.

(3) The inspection authority of the checking list can be justified because it is the present method of protecting passengers, which is compatible with the right of ENTERING of most passengers. Therefore, (not q)=$(q \rightarrow T)=1$ and $q=(T \rightarrow q)=0$.

Figure 4 illustrates the above reasoning, which represents that the potentially conflicting goals of individual rights to ENTERING, which corresponds to R3, and of the mission to protect the passengers, which corresponds to R2, with a coordinator, R1, comprise the same Latin square as Table 1.[10] As observed in the preceding section, changing R1 to R6 does not affect the majority decision. Therefore, it may be safely said that the cognitive system is not malleable in this case for the quarantine story.

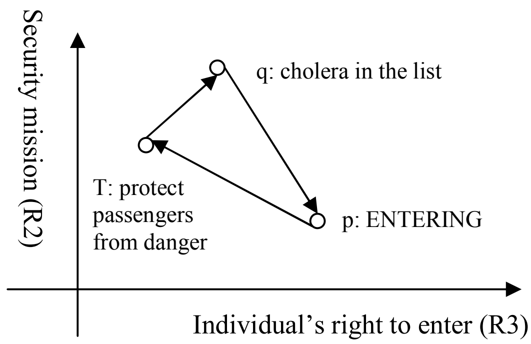

Figure 4: The permission schema as a coordination of conflicting rights.

[7] The notion of malleability I introduced in this paper is similar to, but different from, the conventional notion of *strategic manipulability*, which has been used in social choice theory and game theory (Taylor, 2005). A decision rule is strategically manipulable *iff* a single deviation brings about a relation in favor of the manipulator's individual ordering. And a decision rule is malleable *iff* a unilateral deviation may occur without changing the winner who is the deviator's favorite.

[8] Typically, TA (true antecedent) >TC (true consequence) = FC (false consequence) >FA (false antecedent). See Evans and Over (2004), pp.75-76. This aggregated distribution uses the negation paradigm developed by Evans and Lynch (1973). Two restricted domains 1-2-5-6 and 1-2-3-6 can approximate this. The full empirical frequencies reported by Evans and Lynch (1973) also can be reproduced except for a few nonlinear components such that *p* & *q* & *not q*.

[9] Unlike other domain or population-specific (or thematic) content, such as the Drinking Age Rule, or the Postal Rule (See Evans (1989) p.81, Table 4.3), the Permission Schema does not trivially reduce the WST into a familiar operational task.

[10] Geographically this ordering profile is similar to the asymmetric dominance in the consumer choice context (Simonson, 1992).

8. Concluding remarks

In this paper, I introduced a preference aggregation based modeling for human reasoning, especially for the Wason selection task and its computational realization. Cyclic majority can solve WST normatively, but it is malleable and causes cognitive biases. The model also explains why the permission schema is stable.

Appendix (PROLOG code for WST)

```
aprop(P):-member(P,[p,q,not_p,not_q]).
basic_belief(K):- member(K,[1,2,3]).
extended_belief(K):-
     basic_belief(K);member(K,[4,5,6]).
:- dynamic ':'/2.
% beleif system 1—6 (See Figure 2 and Section 7)

% show all the authorized inspections
to_inspect(N) :-
   extended_belief(N),nl,write(N:' '),
   failed_aprop_ex(N,A),write([A]),fail.
to_inspect(_).
to_inspect :-  basic_belief(N), to_inspect(N), fail.
to_inspect :- majority(basic).
failed_aprop_r(S,N,A) :-
   extended_belief(N), member(N,S),
    aprop(A),\+ r(N):A.
majority(TYPE) :-
   nl,write('M':' '), poll(TYPE,A,_,N),N>=2,
    write([A]), fail.
majority(_).
poll_target(basic,K,A,failed_aprop_r([1,2,3],K,A)).
poll_target(X-Y-Z,
    K,A, failed_aprop_r([X,Y,Z],K,A)).
poll_target(X-Y,K,A,failed_aprop_r([X,Y],K,A)).
poll(TYPE,A,L,N):-
   poll_target(TYPE,K,A,TARGET),
   bagof(K,(TARGET),L), length(L,N).
voters(P):- member(P,[1-2, 1-5, 6-2,
  1-2-3, 1-2-4, 1-5-3, 6-2-3]).

% display beleif systems
write_aprop((r(X):P:-true)):-!,write((r(X):P)).
write_aprop((r(X):P:-Q)):- write((r(X):P:-Q)).
lprop(X):- member(X,[1,2,3,4]),aprop(P),
   clause(r(X):P,Q), nl,
    write_aprop((r(X):P:-Q)), fail.
lprop(_).
lprop:- lprop(_).
lprop.

/*  demo  */
?- majority(basic).
M: [p][not_q]

Yes
?- majority(1-2-4).
M: [p][q]

Yes
?- majority(1-3-5).
M: [p][not_p][not_q]

Yes
?-
```

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