Decisiveness of Decision Maker: A Multiple-Self Method [1]

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Abstract

Nonlinear cumulative probability weighting functions, such as monotone convex / concave capacity, or inverse S shaped function, its curvature changes from concave to convex, have been extensively examined in the recent literature of decision making. Although cumulative representation of separable utility models, such as rank-dependent expected utility (RDEU) or cumulative prospect theory (CPT), are regarded as the quantitative representation of decision maker with uncertain knowledge base (or evidential corpus), it is vulnerable to describe hedging behavior under linear utility weight. In this paper, focusing on goal seeking behavior (i.e., a sort of sequential search process), a two-stage simulation method to elicit and to approximate the nonlinear probability weighting functions with cumulative representation of utility has demonstrated. I interpreted these weights as the decentralized representation of uncertain knowledge of decision maker. The notion of "decisiveness" has the major role to elicit nonlinear probability weight from experimental patterns of dynamic allocation, as the elicited weight to be approximated with consonant beliefs (and these conjugates). Also I demonstrated the computer simulations using spreadsheet models and GA optimization in order to do that end. The experimental data has been collected from the iterative multi-choice test, a web-based experimentation system I developed, where students permit to bet his/ her own initial endowments to judge their choice partially (pJudge) unless the endowments vanish, and used in order to apply these decision models to evaluate imperfect knowledge. An elicited nonlinear probability weight by M vius inversion may be interpreted as a representation of the distributed beliefs of the decision maker who search the items sequentially. In additon, I found a strong correlation between the pJuges and the "regret index" which is the ratio of subjective disachievement and suquare root of difficulty.

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[1] This paper is an extended abstract of its full version K. Indo(2002). Decisiveness of decision maker : a multiple self method. Research Bulletin of Economics, Kanto Gakuen University 29/1, 29-62. And an earlier draft of this paper has delivered in the 5th conference on experimental economics at Hakodate Future University, 29 Aug 2001.

1. Introduction

In real life, usually a decision maker is complex cognitive-emotional system with conflicting goals and imperfect knowledge about the future courses of action by itself or by others. Sometimes there is neither perfect satisfaction nor regret-free result of attainable (i.e., the best), and therefore we must be dealing with going concern. In other words, because misspecifications of problem and unforeseen contingencies are inevitable in our cognitive-social life and also we aware of it, some reliable prescriptive way to repair our irrationality, besides rational choice theories, such as expected utility theory and game theory, cognitive psychology, management science, or information systems which support decision makers, are needed.

In this regard, it seems me that information acquisition by mixture of knowledge and search with subtle objectives satisfying us by means of making mental representations (i.e., "editing" and "framing" in terms of Kahneman and Tversky (1979)), as well as economic activities with concrete objectives by means of monetary transaction, is noteworthy and is stimulus of inquiry because of its potential to improve irrationality not only with normative lecturing ("Should be rational!"), but with prescriptive way to design remedy against it.

Multiple-self, that regards a single decision maker, who usually tend to violate expected utility theory, as a collection of agents, and optionally with a game theoretic mechanism of coordination, is probably one of the most appealing descriptive ways, at least intuitively, in order to model these properties pertain to bounded rational decision makers. Precommitment, the notion repeatedly mentioned in the literature, the personal rules of various constraints against decision maker's weak will power, in order to remedy conflicting mind. The analogy of bargaining game in the economics of self-control (Schelling, 1986), the analogy of reputation game in the model of addiction (Ainslie, 1992), as well as the dynamic inconsistency of nonexpected utility maximizer (Stortz, 1956). Mental account (Thaler, 1990) is another type of precommitment mechanism, the personal rules to manage money in distinctive budgets. Shefrin (1998) reviewed these models with respect to the hyperbolic discounting thoroughly. Elster (1986) collected several essayes in this field.

My proposal is a two-stage method to elicit cumulative probability weight of decision makers in sequential choice, and to translate it into (possibly decentralized) uncertain knowledge systems by linear combination of approximated consonant belief function and its conjugate plausibility function. In first-stage of this method, the notion of decisiveness, which can be regarded as an analogy of precommitment in multiple-self models, plays the role of keystone to elicit probability weight in accordance with counterfactual reasoning of decision makers.

In this paper, focusing on the situation of experimental multi-choice test I developed, a practical application of our theory to computer-aided web learning on Internet, where students permit to bet his/her own initial endowments to judge their choice partially ("pJudge" for short) unless the endowments vanish, I tried to apply and verify these decision models to evaluate imperfect knowledge base of my students via computational method. Experimental data has collected in 24 Apr and during 25 Apr to 2 May 2001 of students of my two classes. In our approach, it is assumed that decision makers (i.e., students of my class) have nearly "consonant" beliefs about when the right answer will be found, i.e., the stopping time of search process to find the right answer of each question with 5 branches respectively.

Consonant beliefs are beliefs of events such that there are only nested focal elements have positive basic probability mass (bpa). M bius inversion is the tool for computing the bpa of belief function. It is also familiar to students of cooperative game theory and multi-criteria decision making (i.e., the Shapley-Banzhaf indices which measure marginal contribution of agent to coalition). Approximated beliefs may remain some small positive mass out of nested monotone (convex) beliefs. With a little surprise, it may be usefull to approximate inverse S shaped cumulative weight with linear combination of its conjugate function by means of M bius inversion.

Another point of view which may draw attention of researchers to the cognitive aspect of this sequential choice problem is the updating rule for the Choquet capacity or the nonlinear probability weight (Cohen, Gilboa, Jaffray, and Schmeidler, 1999). The choice of decision maker who has multiple prior beliefs and obeys Hurwicz criteria also can be explained by the CPT model with a linear combination of convex capacity and its conjugate, and its dynamic counterpart, updating rule will reflects its optimism/pessimsm parameter. Therefore the observation data from well-designed sequential choice problem may reveal this model parameter too.

I will report with the some experimental data, and simulation results using spreadsheet models and optimization tools. These tools a re familiar to both academic users and business decision makers. The genetic algorithm (GA) to realize the intelligent systems, by means of simulating evolutionary process of gene, has pioneered by John Holland at first in the early 1960s (Mitchell, 1996). GA optimization is a versatile tool for many real applications where we have to formalize and to solve nonlinear otimization problems in time, at least approximately.

The remainder of this paper follows: In the next 2 sections, related works with the notion of decisiveness including multiple-self, Choquet representation, belief function, and probability weighting function are briefly reviewed (section 3 has omitted in this version). In setion 4, I introduce the iterative multi-choice test system to be analyzed. In section 5, the main part of this paper, the experimental data and the "decisiveness-based" algorithm of elicitation-approximation procedure for cumulative representation under nonlinear probability weighting function thereby to elicit a distributed knowledge base will be argued. (Only a part of the section has demonstrated in this version.)

2. Related works and motivational background

The idea of game theoretic formulation for multiple-self decision maker is not new in the literature on non-expected utility theory (Strotz, 1956; Karni and Safra, 1989, 1990). And recently absentmindedness of game player with imperfect recall has draw attention of many game theorists. But there seems no experimental validation, epistemic foundation, and inductive modeling method of such theories, except for time-preference models in distributed (intertemporal) choice context and Voter's illusion type experimental study in cognitive psychology.

Karni and Safra (1990) proposed the notion of behavioral consistency, distributed agent representation of RDEU nonexpected utility maximizer to apply it to the models of auctions and search. As for search model, they found the analog of reservation price property under quasi-convex utility functional that is decisive as EU maximizer, and the existence of upper-lower interval of indecisive stopping strategy under quasi-concavity. In their FIG. 1, p.395, they showed an example of a behaviorally inconsistent decentralized search tree. In spite of their insistence, it seems natural to me that the agent passes the act "a" to get middle level outcome with the certainty up at the first choice node, because of the chance of same or better "with certainty" remains until the sub-node 1 has played. That is somewhat similar advantage to EU maximizer's sequential search (Weitzman, 1979), and preference for flexibility (Nehring, 1999).

Besides traditional ego psychology and social psychology, "society of mind" (Minsky, 1986), one of the most stimulative paradigm in recent artificial intelligence research has similar features of multiple-self decision model except for its commitment on the reduction of real intelligent system into relative simpler units and administrative functions of their interaction. Piaget's "cognitive equilibrium", or Simon's "neally decomposable systems" are also considered as other lineage of multiple-self.

By the way, in belief-based modeling techniques, both cumulative representations of utility (rank-dependent expected utility (RDEU) or cumulative prospect theory (CPT)) and inverse S shaped probability weight, have been experimentally examined by recent decision science researchers. Especially, capacities (i.e., nonadditive probability weight) of Choquet integral and its relation to the max-min utility representation under multi-probabilities (Gilboa and Schmeidler, 1994; Mukerji, 1997) in these models are utilized to model decision maker's attitude towards uncertainty in probability, or so called ambiguity aversion (under convex capacities and belief functions) or ambiguity seeking (under concave capacities and plausibility functions).

Cumulative representation of separable utility models with nonadditive probabilities or ambiguous beliefs has developed by Quiggin, Schmeidler, Gilboa, Yaari, and many other contributed researchers. In papers on axiomitization of this sort of representation, usually replace independence axiom with comonotonic independence (i.e., ordinal independence) and it does not violate to stochastic dominance. As for decision theory, nonadditive probability models was intended to model ambiguous beliefs and resolve Ellsberg's paradox (Ellsberg, 1961) an apparent contradiction to Savage's Subjective EU, at first, then apply to the game theory, portfolio theory and so on (Dow and Werlang, 1992ab, 1994). But it can also explain besides other type of stylized violations to EU as well as Allais padadox and preference reversal, those cases which are cannot explained by monotone capacities (Segal, 1987; Karni and Safra, 1987).

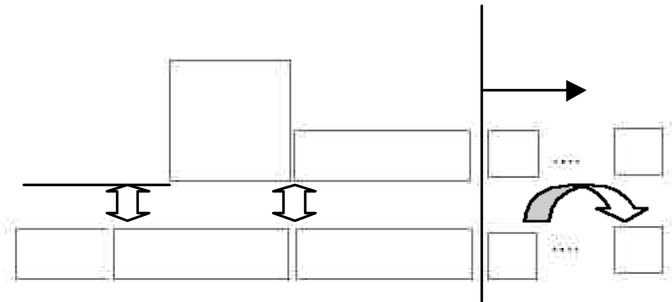
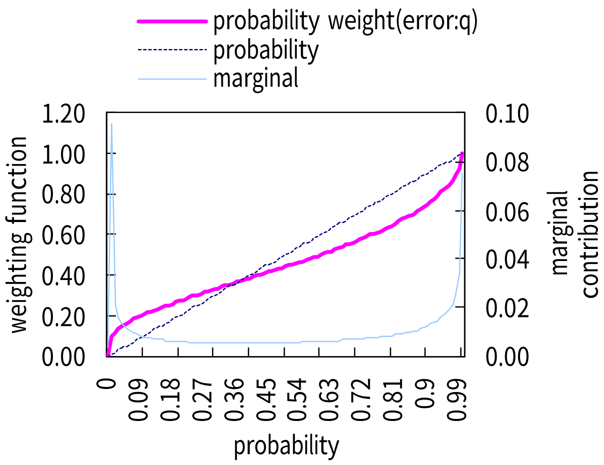


Figure 2.1 Intuitive illustration of preference ladder of Wu and Gonzalez (1996).

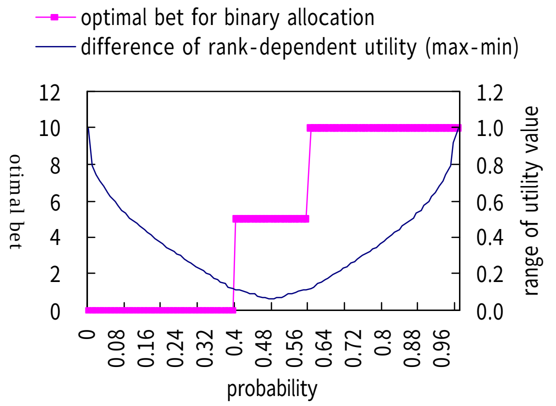
Figure 2.1 illustrates how the shift of curvature of weight concave to convex in probability affects attitude of decision maker's risk preference changing from optimistic to pessimistic. Each rectangle box represents an event or a lottery, and its hight and width correspond to the prize and probability of that lottery. Wu and Gonzalez utilized the fact that, by adding the sequence of same common consequences to a risky gamble and its equivalent but safety gamble, tendency to choose risky is increasing then decreasing. This is called "preference ladder" applied to nonparametric elicitation technique for inverse S shape weight (Wu and Gonzalez, 1996).

Recently, George Wu linked iterative RDEU model and nonlinear probability weight to decision maker's "anxiety" (or thought time based intensity), and it generalizes Bell's anticipated regret (Wu, 1999). Wu's anxiety model was intended to incorporate the process of allocating cognitive resources, i.e., attention, and implicitly interestingness, into the decision models. (I like to call this as "internal search", vs. external standard one, which can be regarded as the cognitive process of decision maker in the intelligence and design activities (Simon, 1996). Shafer and Tversky (1985) also pined down the design process of explorative statistical reasoning contrasting Bayes rule and belief functions.)

Roughly speaking, concavity in probability weight affects RDEU optimizers to have tendency to delay their decisive timing of choice. Relation between concavity / convexity in probability and preference for delayed resolution / for early resolution has been observed by researchers who are dealing with distinct models with or without rank-dependent representation (Karni and Safra, 1990; Nerhring, 1999; Grant et al., 1998). Because of monotone convex / concave and inverse S shaped weighting functions can represent only with 1 or 2 attention peaks as for probability, and RDEU cumulate in accordance with rank of outcomes by its standard utility, I guess that they could have expanded the idea by Dempster-Shafer's theory, assuming the redistribution of probability mass in belief system represents it, in the iterative multi-choice test with uncertain knowledge base.



(a)



(b)

Figure 2.2 Prelec's probability weighting function Exp(-(-Ln p)^alpha) when alpha=0.55 (a) and the optimal binary allocation which maximizes rank-dependent expected value (b).

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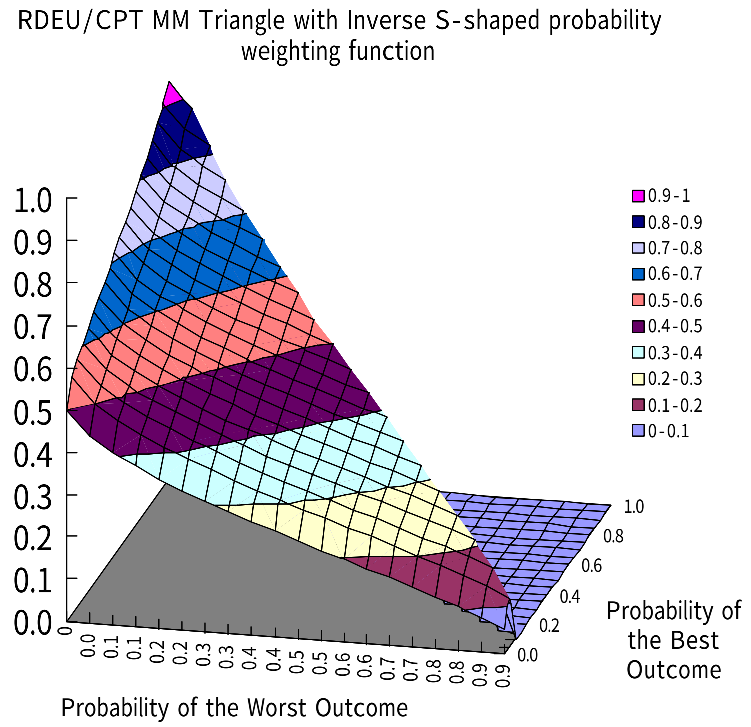


Figure 2.3 3-D visualization of the Marshak-Machina triangle for cumulative representations (RDEU with linear utility and decision weight Exp(-(-Ln p)^0.7).)

Inverse S shaped probability weight (Wu and Gonzalez, 1996, 1999; Prelec, 1998; Fox and Tversky, 1998; Tversky and Wakker, 1995) shows "from concave to convex" property at probability 0.3 - 0.4 (Figure 2.2), and it can explain two types of marginal effect: the certainty effect, tendency to under evaluate probability near to 1 and the possibility effect, tendency to over evaluate probability near to 0. Prospect theory of Tversky and Kahneman in risk situations (Kahneman and Tversky, 1979) has extended (including to uncertainty situation) with this type of weight that shows bounded subadditivity Tversky and his collaborators are insisted (i.e., Support Theory), and distinctive weights for gains and losses (Tversky and Kahneman, 1993; Wakker and Tversky, 1993). Recently, new elicitation techniques for biased utility and nonlinear probability weight, with or without the standard sequence, have developed (Abdellaoui, 2000; Bleichrodt and Pianto, 2000; Wakker and Deneffe, 1996).

However, up to middle 1990s, there are several experimental studies reported in 1990's that researchers did not appreciated descriptive improvement of this type of models against expected utility and other alternative models for risk attitude, and its advantage is limited only to the marginal of Marschak-Machina triangle (Wakker, Erev and Weber, 1994; Wakker, Mangelsdorff and Weber, 1994; Harless and Camerer, 1994; Hey and Orme, 1994). It has observed that comonotonic Independence rather tends to be violated (Wu, 1994; Fennema and Wakker, 1996).

Additionally, it cannot explicate various hedging behavior without biased utility, even if augmented with the inverse S-shaped probability weight (see Figure 2.2 and Figure 5.6). Despite of valunerability in descriptive power, it has not only a normatively, but cognitively appealing character. By using techniques of Dempster-Shafer theory (Mukerji, 1997; Mongin, 1994; Jaffrey and Wakker, 1994), the cumulative representation of separable utility models can be regarded as the quantitative representation of uncertain knowledge base (or evidential corpus (Smets, 1998)) of bounded rational decisioin maker.

I also propose the notion of decisiveness with approximated consonant belief models so as to interpret RDEU and decision weights. Kahneman and Varey (1997) has stated the notion of "decisiveness" as exclusive event, in relation to the notion of "propensities" that is the psychological counterpart to probability, which is not accordance with probability in human judgment, such as a counterfactual statement "He almost won". The notion of decisiveness by Kahneman and Varey has similar nature to the "minimality" principle, in the sense of Ramsey test and "epistemic entrenchment" in belief revision theory (G rdenfors, 1986) studied by philosophers of language and decision theorists, that the meaning of counterfactual sentence as the "nearest impossible possible world" from the true (or current) possible state of the world (Lewis, 1976). In this paper, the two-stage elicitation-approximation procedure I proposed incorporates it in the first stage, which measures minimal distance of experimental data pattern from the optimal pattern of RDEU maximization as this idea of "nearest impossible possible world". And also in the first stage, this measurement of distance from RDEU-optimaility ranks the series of possible patterns of behavior.

As explained in the next section, the axiomatization of epistemic entrenchment is identical to consonant belief function, and so as to necessity measure. So, readers may infer that the decisive-critical pair of events may be translated into belief function and plausibility function pair, or lower and upper envelopes of set of probabilities. But I suspect that it is meaningless until this analogy be related to inverse S shaped weight by its approximation procedure.

3. Review of cumulative representations and belief functions

<Omitted>

4. Iterative multi-choice test: A web-based examination system

The iterative multiple-choice test is a web-based experimental examination system where the answer of students can be partially marked (partial judge, or pJudge) with self-allotment of points within endowment (the quota, for each question, 10 points, initially). This exam system is accessible from anywhere that has a connection to WWW of Internet (URL http://www.us.kanto-gakuen.ac.jp/kindo/). The program code is written in JavaScript and Html. CGI (Common Gateway Interface) has used minimally so that students can submit the experimental data by the exam system. The experimental data submitted by students include their answers, allotments, judged results, score, questionnaires for both of their subjective report of difficulty and performance about each question, timing of choice for selected options and partial judgment, and some other questionnaires about student ' s objective attributes and opinions.

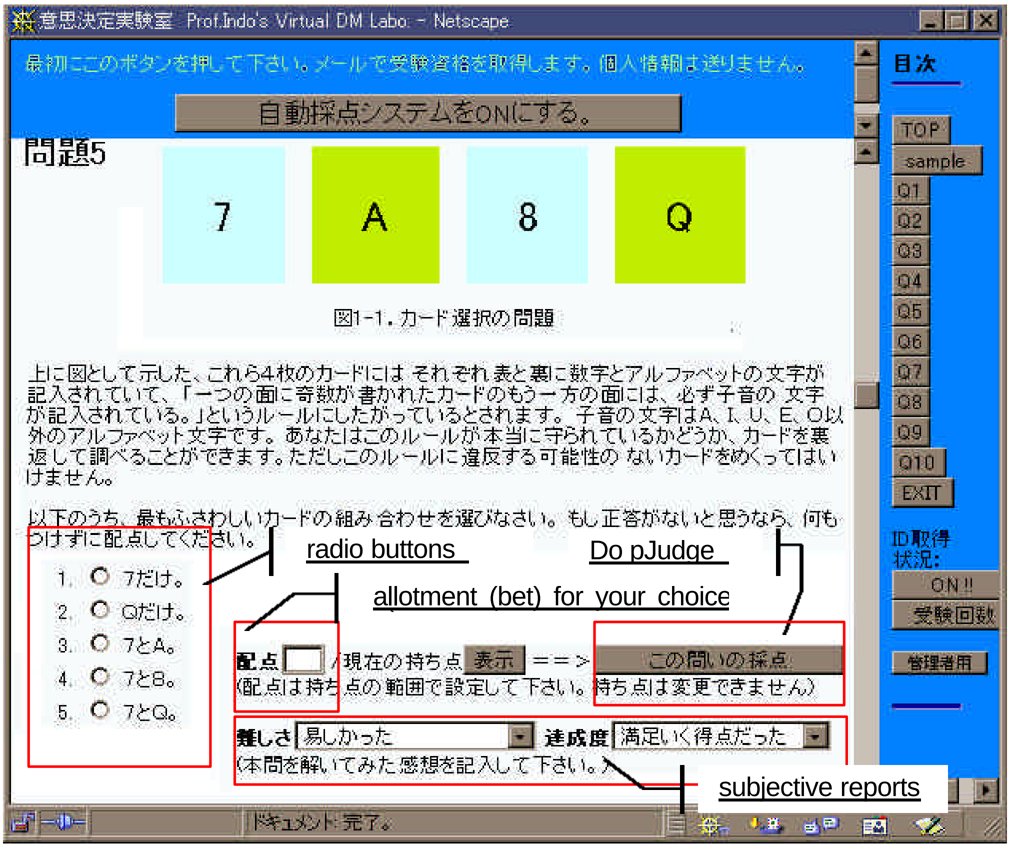


Figure 4.1 A sample problem (Problem Q5, a version of Wason ' s selection task.)

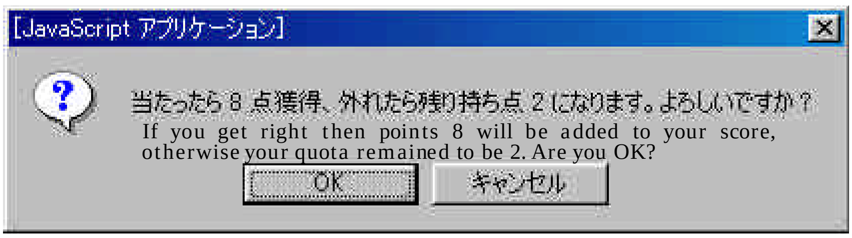


Figure 4.2 Confirming window to pJudge as a lottery.

The examination is consists of 10 questions. Each question has a description of problem and has 5 radio buttons each of which is an alternative answer for that question respectively. The language is Japanese only. During the iteration of multi-choice trials, examinee (i.e., subject) is permitted to bet a portion of his/her own points freely to one of the alternatives that has selected within the quoata (i.e., initially, 10 points), and the system marks it in background process.

If the examinee wants to know whether or not the chosen answer is correct, he /she may ask the system by means of submit button is permitted. If it is correct, the question is completed, and the allotted points are added to the score. Also, he/she can re-challenge when it is not correct, unless the remainder points vanish after decreased by the allotment of points.

By the way, if the correct answer exists, anybody can find it by means of the blind search. So it might be the case, even if the examinee has no knowledge about the problem. However, the system marks off wrong answer submitted. Since submission of wrong answer reduces the points allotted from the quoata of the examinee as for that question, and therefore the total score of examinee. Therefore, the betting strategy of serious examinee who has, at least, a partial knowledge about that question is estimated as that optimises the allotment of the point according to the trust about the probability of getting right answer and his own knowledge with respect to that question. Because the checking of a radio button on browser can be made at most 1 item (single option) in the once about each submit for each question, the resulting information structure of the search activity of the examinee, that consists of the repetition of partial marking (i.e., pJudge), is a nesting (or list) with iteratively contrasted the focus-and-remainder pair (as those of "list" denotation in Prolog, [Focus | Remainder] = [Focus, R1, R2, …]). We tried to approximate this by the consonant belief function (cf., section 3) the each focus element of which is finding a correct answer up to the k-th pJudge.

The search strategy of the examnee is to define the possible reward function (i.e., payoffs) for the search activity by him/her-self, and then it will be an example of multiple self model with a prescriptive role which the decision-maker itself play as the agent by of the successive search problem which has designed by itself. Also, via control of the allotment, and therefore of the stopping time, the decisiveness of the examinee reflects the prospected ranking at that time, which indirectly affects in the future choice behaviour and allotment of points. For example, supposing a na vestrategy that if an alternative 1 is the highest prospective of the examinee, he/she will allots the all point remainder to it. This is totally rational for EV-maximizer.

However, if the knowledge base is not decisive, psychological state of mind, such as regret or disappointment, with some degree, may affet and to depress those optimism because of the expectation of failure (i.e., risk). Also, it is possible to expect that the increase in anxiety and regret diminishes the allotment gradually when the decisiveness of the knowledge is low. As for the regret, our system provides the questionnaire method to record the subjective performance and difficulty enter from the pull-down menu. By computing weighted averages of "difficulty" (d1) and "disachievement" (d2) for each question, I found that "regret index" (RI = d2 / sqrt(d1)) has strong correlatin to the number of pJudges (see Figure 5.2).

In addtition to above, in this examination, the possibility that no correct answer in each question is suggested in the earlier part of this examination. When a right answer is lacked the allotment points will scored if no checked. But if the radio button for a question has checked once, although it is possible to change the checked, user cannot return it to the initial state where no button has checked. Although this becomes a penalty to the choice without deliberation or consideration, the possibility that the examinee doesn't notice that possibility of no right answer pertains to "unforeseen contingencies".

5. Experimental data and simulation results

This section provides the summary results of our experimentation of iterative multi-choice test with 5 branches (5-taku in Japanese) where students are permitted to allocate endowments (i.e., 10 points, initially) to bet for his / her choice.

Experimental data and model

The experimental data to be analyzed are of 2 (+1) class, total 24 students' submitted results (ID 8--31). The 7 cases (ID 8--14) are results of Experiment A time-controlled in my class about 1 hour. The 3 cases (ID 15--17) are also categorized in A since both monitored but ended within 10 minutes. The remainder 14 cases (ID 18--31) are results of Experiment B under free-submit condition via Internet. The 7 cases we collected previously (ID 1--7) are excluded because it was same exam system but almost different problems. (Tables 5.1--5.4, Figures 5.1--5.3)

And we show also some simulation results for simple 2-choice test (2-taku in Japanese) assuming RDEU /CPT-maximization (Figure 5.4). In the case of CPT, we assumed linear utility. Then the elicitation-approximation procedure will be demonstrated as follows. Firstly, to determine curvature of probability weighting function, given a sequence of bet X, ex. X=[2,4,2,1,1,0], the probability weighting function which has the minimal decisiveness measure, i.e., the smallest penalty weight under which the observed sequential allocation pattern is optimal has to be found (Figures 5.8 5.11). Secondly, to find the approximation of this cumulative probability weight, a linear combination of a (neally) consonant belief function and its conjugate plausibility function has to be found (Fifures 5.6 5.7). Then probability masses via inversion represent a solution for decision maker's (decentralized) cognitive resource allocation (i.e., attention) problem.

Table 5.1 Partial judges and the improvement (except for two objective questionnaires)

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total
%right@1st	71%	-	-	33%	50%	38%	25%	13%	42%	42%	39%
%right@fin	88%	-	-	58%	75%	75%	58%	71%	67%	75%	71%
%improved	17%	-	-	25%	25%	38%	33%	58%	25%	33%	32%
mean #pJudge	1.3	1.0	1.0	1.7	1.4	1.4	1.8	1.4	1.4	1.7	1.4

Table 5.2 Total time consumptions in experiment A (time controlled) and B (free submit)

	average	SD	Max	Min
experiment A	0:43:46	0:07:55	0:56:12	0:35:18
experiment B	0:29:34	0:36:07	2:26:40	0:03:43

Table 5.3 Reported subjective difficulty and reported subjective performance

(a) difficulty reported

reported difficulty

	sample	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
#A	21	15	9	7	6	4	8	8	11	10	8
#B	1	5	3	8	4	6	2	4	1	5	0
#C	1	4	10	7	7	8	11	9	8	6	13
#D	1	0	2	2	7	6	3	3	4	3	3
#E	0	0	0	0	0	0	0	0	0	0	0
SUM	24	24	24	24	24	24	24	24	24	24	24
	sample	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
%A	88%	79%	79%	75%	42%	42%	50%	50%	58%	58%	54%
%B	8%	4%	8%	17%	25%	29%	29%	13%	8%	25%	17%
%C	4%	8%	8%	4%	13%	13%	4%	17%	25%	8%	13%
%D	0%	8%	0%	4%	21%	13%	17%	17%	8%	4%	17%
%E	0%	0%	4%	0%	0%	4%	0%	4%	0%	4%	0%
SUM	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

(a) achievement reported

reported achievement

	sample	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
#A	21	19	19	18	10	10	12	12	14	14	13
#B	2	1	2	4	6	7	7	3	2	6	4
#C	1	2	2	1	3	3	1	4	6	2	3
#D	0	2	0	1	5	3	4	4	2	1	4
#E	0	0	1	0	0	1	0	1	0	1	0
SUM	24	24	24	24	24	24	24	24	24	24	24
	sample	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
%A	88%	79%	79%	75%	42%	42%	50%	50%	58%	58%	54%
%B	8%	4%	8%	17%	25%	29%	29%	13%	8%	25%	17%
%C	4%	8%	8%	4%	13%	13%	4%	17%	25%	8%	13%
%D	0%	8%	0%	4%	21%	13%	17%	17%	8%	4%	17%
%E	0%	0%	4%	0%	0%	4%	0%	4%	0%	4%	0%
SUM	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Table 5.4(a) The number of trials for partial judges and time consumption data

Number of pJudges for each question

cells colored are larger between subjects within subject both

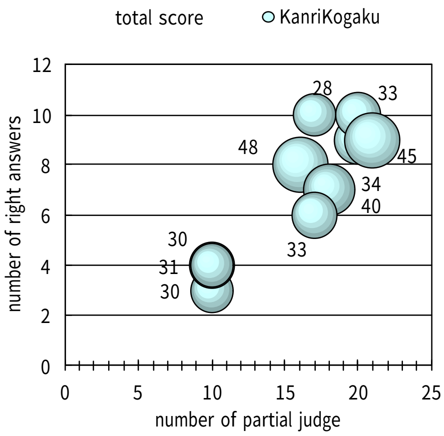
student	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Average	SD	MAX	total
1	3	1	5	1	0	0	0	0	0	0	1	1.7	5	10
2	2	5	0	0	0	0	0	0	0	0	1	1.6	5	7
3	6	0	0	0	0	0	0	0	0	0	1	1.9	6	6
4	0	1	2	2	2	2	5	0	0	0	1	1.6	5	14
5	0	0	0	0	0	0	0	0	0	0	0	0.0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0.0	0	0
7	1	5	7	2	1	1	1	1	1	1	2	2.1	7	21
8	3	1	1	3	1	1	1	1	1	3	2	1.0	3	16
9	2	1	1	1	1	1	3	3	3	2	2	0.9	3	18
10	1	1	1	4	1	1	2	3	2	1	2	1.1	4	17
11	2	1	1	4	4	4	4	1	1	1	2	1.5	4	23
12	3	1	1	2	1	1	3	2	1	2	2	0.8	3	17
13	1	1	1	2	3	3	2	2	1	2	2	0.8	3	18
14	1	1	1	5	2	2	3	2	2	3	2	1.2	5	22
15	1	1	1	1	1	1	1	1	1	1	1	0.0	1	10
16	1	1	1	1	1	1	1	1	1	1	1	0.0	1	10
17	1	1	1	1	1	1	1	1	1	1	1	0.0	1	10
18	1	1	1	1	1	1	1	1	1	1	1	0.0	1	10
19	1	1	1	2	1	1	2	1	1	1	1	0.4	2	12
20	1	1	1	1	1	1	1	2	1	1	1	0.3	2	11
21	1	1	1	1	1	1	2	1	4	4	2	1.3	4	17
22	1	1	1	2	2	2	3	1	1	3	2	0.8	3	17
23	1	1	1	1	2	2	3	2	3	3	2	0.9	3	19
24	1	1	1	1	1	1	1	1	1	1	1	0.0	1	10
25	1	1	1	1	1	1	3	2	1	3	2	0.8	3	15
26	1	1	1	1	1	1	1	1	1	1	1	0.0	1	10
27	1	1	1	1	1	1	1	1	1	1	1	0.0	1	10
28	1	1	1	1	2	2	1	1	1	1	1	0.4	2	12
29	1	1	1	1	1	1	1	1	1	1	1	0.0	1	10
30	1	1	1	1	1	1	1	1	1	1	1	0.0	1	10
31	1	1	1	1	1	1	1	1	1	1	1	0.0	1	10
total	30	24	24	40	33	33	43	34	33	40	33.4		7	392
Average	1	1	1	2	1	1	2	1	1	2	1			
SD	1	0	0	1	1	1	1	1	1	1	1			
MAX	3	1	1	5	4	4	4	3	4	4	5			
(expect for 1-7)														

Table 5.4(b) The number of trials for partial judges and time consumption data

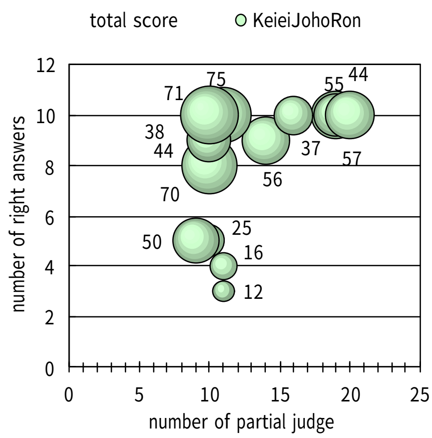
Total time consumptions

cells colored are larger between subjects within subject both

student	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Average	SD
1	0:02:17	0:00:40	0:01:04	0:00:28	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:27	0:00:45
2	0:12:31	0:04:25	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:01:42	0:04:03
3	0:08:26	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:51	0:02:40
4	0:00:00	0:00:23	0:00:21	0:00:23	0:00:24	0:00:21	0:00:35	0:00:00	0:00:00	0:00:00	0:00:15	0:00:13
5	0:01:18	0:01:05	0:00:11	0:00:47	0:00:31	0:00:15	0:00:11	0:00:12	0:00:15	0:00:13	0:00:30	0:00:25
6	0:01:21	0:01:30	0:00:06	0:00:04	0:00:11	0:00:00	0:00:15	0:00:00	0:00:00	0:00:00	0:00:21	0:00:35
7	0:05:33	0:01:53	0:01:31	0:00:36	0:00:42	0:00:26	0:00:28	0:00:24	0:00:21	0:00:25	0:01:14	0:01:36
8	0:02:36	0:07:08	0:01:27	0:08:32	0:04:19	0:09:31	0:06:20	0:03:19	0:04:01	0:05:17	0:05:15	0:02:37
9	0:02:06	0:04:44	0:00:39	0:03:43	0:01:57	0:04:29	0:06:06	0:03:34	0:03:20	0:04:53	0:03:33	0:01:37
10	0:05:45	0:03:40	0:02:39	0:08:45	0:04:45	0:08:14	0:04:22	0:04:16	0:04:32	0:01:39	0:04:52	0:02:14
11	0:01:45	0:03:56	0:01:08	0:03:42	0:02:52	0:07:36	0:03:48	0:03:15	0:03:06	0:04:39	0:03:35	0:01:45
12	0:01:47	0:04:36	0:01:34	0:01:52	0:02:23	0:02:55	0:02:37	0:01:47	0:02:10	0:01:36	0:02:20	0:00:55
13	0:00:59	0:02:12	0:02:49	0:03:19	0:02:01	0:02:47	0:02:43	0:02:26	0:01:34	0:04:07	0:02:30	0:00:53
14	0:03:54	0:03:44	0:01:11	0:04:12	0:02:51	0:02:09	0:03:06	0:01:25	0:02:13	0:04:19	0:02:54	0:01:08
15	0:00:41	0:00:38	0:00:11	0:00:31	0:02:07	0:04:11	0:00:37	0:00:15	0:00:13	0:00:12	0:00:58	0:01:16
16	0:00:20	0:00:25	0:00:32	0:00:14	0:00:30	0:00:34	0:00:26	0:00:20	0:00:40	0:00:18	0:00:26	0:00:08
17	0:00:21	0:00:37	0:00:23	0:00:07	0:01:38	0:00:08	0:00:18	0:00:21	0:00:34	0:00:29	0:00:30	0:00:26
18	0:00:52	0:00:17	0:00:13	0:00:09	0:00:17	0:00:36	0:00:53	0:00:15	0:00:23	0:15:01	0:01:54	0:04:37
19	0:14:56	0:05:20	0:04:21	0:22:20	0:24:31	0:25:42	0:11:49	0:18:04	0:11:58	0:07:39	0:14:40	0:07:48
20	0:05:44	0:00:33	0:00:33	0:00:58	0:00:19	0:00:21	0:01:20	0:00:39	0:00:29	0:05:02	0:01:36	0:02:02
21	0:01:18	0:01:07	0:00:11	0:00:24	0:03:06	0:05:41	0:04:46	0:00:43	0:02:27	0:03:21	0:02:18	0:01:54
22	0:00:14	0:00:13	0:00:15	0:00:20	0:00:24	0:00:36	0:00:33	0:00:23	0:00:11	0:00:32	0:00:22	0:00:09
23	0:03:03	0:00:44	0:00:39	0:02:12	0:03:13	0:05:26	0:02:17	0:04:16	0:02:09	0:02:14	0:02:37	0:01:28
24	0:01:54	0:00:46	0:01:33	0:01:07	0:02:19	0:01:17	0:00:18	0:00:13	0:00:12	0:00:14	0:00:59	0:00:46
25	0:00:52	0:02:36	0:00:51	0:02:33	0:01:46	0:02:45	0:02:23	0:01:45	0:01:07	0:02:53	0:01:57	0:00:47
26	0:01:13	0:01:12	0:01:36	0:00:29	0:00:26	0:00:33	0:00:25	0:00:25	0:00:24	0:00:29	0:00:43	0:00:27
27	0:00:33	0:00:11	0:00:14	0:00:11	0:00:10	0:00:00	0:00:15	0:00:15	0:00:26	0:00:35	0:00:17	0:00:11
28	0:00:18	0:00:20	0:00:10	0:00:13	0:00:31	0:00:14	0:02:06	0:00:17	0:00:13	0:00:14	0:00:28	0:00:35
29	0:03:51	0:00:52	0:00:30	0:01:06	0:00:29	0:00:35	0:01:13	0:00:43	0:00:43	0:00:36	0:01:04	0:01:01
30	0:01:38	0:01:05	0:00:23	0:00:58	0:00:19	0:00:42	0:01:25	0:00:46	0:01:34	0:00:44	0:00:57	0:00:28
31	0:07:43	0:02:18	0:01:10	0:00:52	0:00:43	0:01:18	0:00:46	0:00:25	0:00:34	0:00:26	0:01:37	0:02:13
total	0.04471	0.03419	0.0175	0.04779	0.0444	0.06134	0.04227	0.0348	0.0314	0.04686	0.04053	
Average	0:02:41	0:02:03	0:01:03	0:02:52	0:02:40	0:03:41	0:02:32	0:02:05	0:01:53	0:02:49	0:02:26	
SD	0:03:16	0:01:58	0:01:01	0:04:47	0:04:51	0:05:27	0:02:41	0:03:40	0:02:30	0:03:22	0:03:21	
(expect for 1-7)												



(a) experiment A: time controlled



(b) experiment B: free submit

Figure 5.1 Total score for each of two conditions

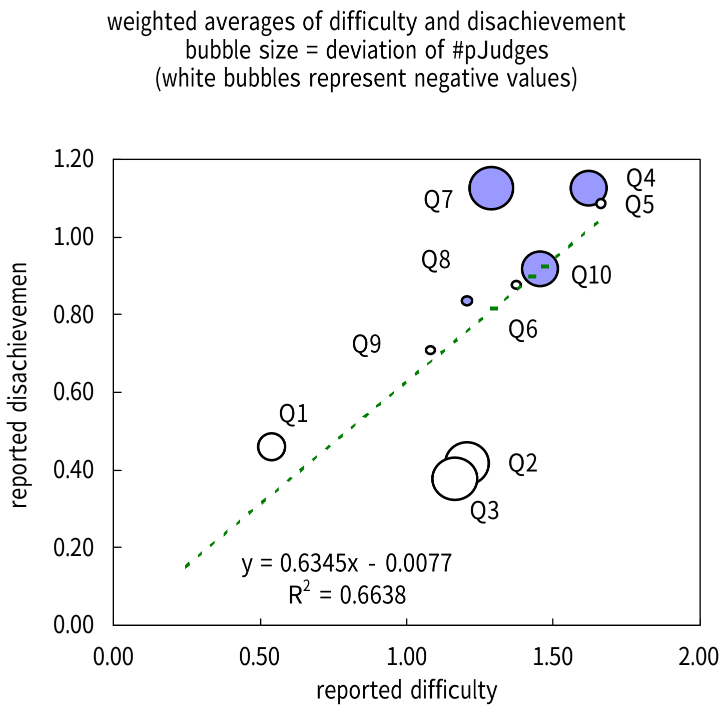


Figure 5.2(a) Weighted averages of "difficulty" and "disachievement"

(A=0; B=1; C=2; D=3; E=4)

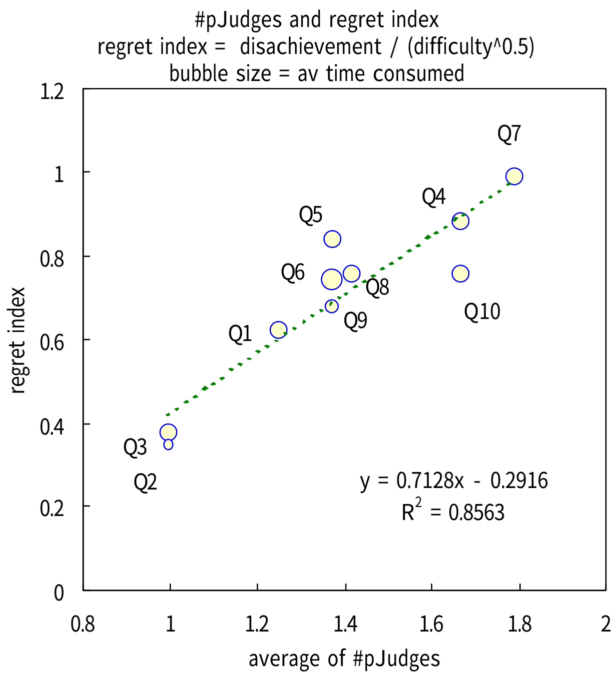


Figure 5.2(b) "Regret index" which has strong correlatin to the number of pJudges

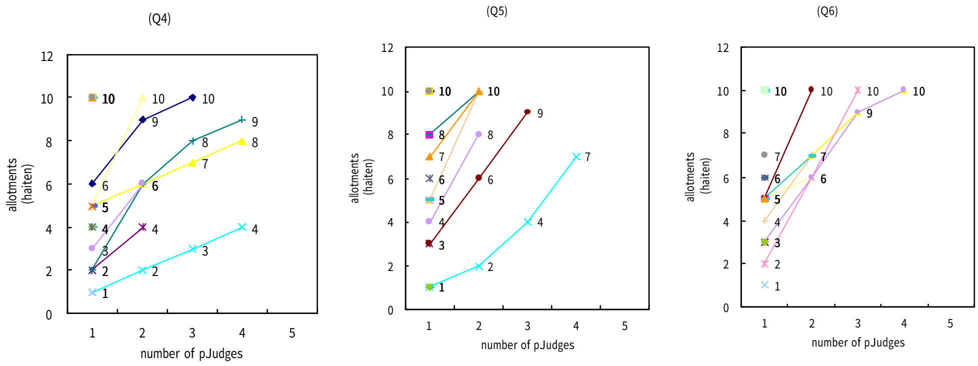


Figure 5.3 (a) Cumulative allocation patterns (Q4-6).

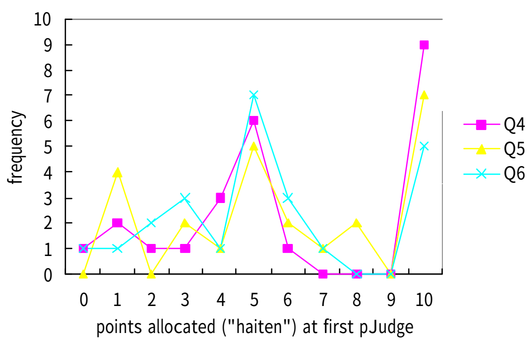
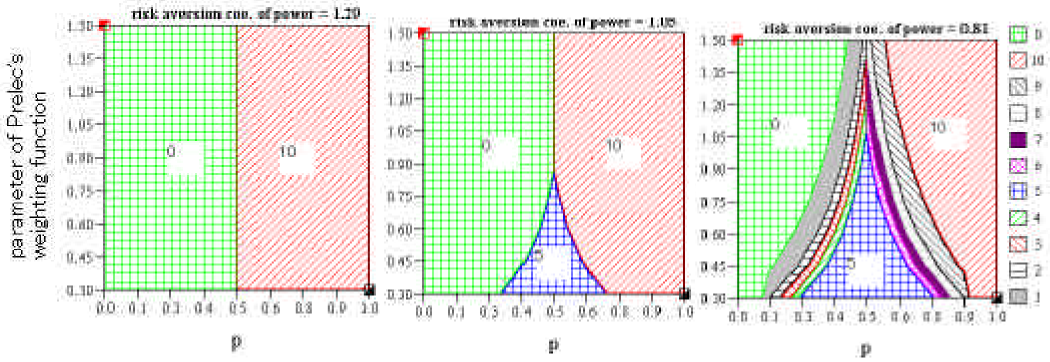


Figure 5.3 (b) Initial allocation pattern in experimental data (Q4~6) .



(a) risk seeking (b) almost risk neutral (c) risk averse

Figure 5.4 Sensitivity analysis of 2-choice test simulation of RDEU/CPT with power utility

Simulation results

Next, let us turn to describe the two-step procedure in order to elicit-and-approximate the nonlinear weighting function of decision maker in the iterative multi-choice examination. Before I explain about our procedure, it may be noteworthy that in the case of risk neutral or risk seeking (i.e., index of power utility≧1), by analogical reasoning from the sensitivity analysis (using TreeAge ' s DATA3.5) for the simple 2-choice problem for an RDEU-maximizer, of course, who has additive probability weights in figure 5.4(a) and (b), we "can" predict (1) at most one intermediate level of allotment is possible, (2) it is exactly brought about at the indifference of rank order, and (3) the corresponding interval of probabilities is decreasing in the single parameter of Prelec ' s function.

In the case of risk averse, such as what you can see in Figure 5.4(c), in this case we conclude that the various levels of partial allotment may occure and there is no hope to determine the curvature. But in our problem, it seems me that the lineality in utility is plausible as for this experimental examination, since the final score is not so "big money" (such as St. Petersberg paradox).

At first, we assume RDEU/CPU maximization with respect to nonlinear probability weights. Additionally we will assume risk neutrality. Then in order to determine its curvature, typically the inverse S shaped as you can see in Figure 5.8, we need to find both a single parameter of the weighting function and a probability distribution over the alternatives. Naturally, we can predict the most likely weighting function is of the smallest discrepancy between utilities of the observed pattern and of the optimal respectively. But the pair of a uniformly distributed masses and a linear weighting function (i.e., additive probability) justifies any pattern of allotments, and so discrepancy vanishes. Indeed the GA optimization process tends to converge to the uniform distribution as in Figure 5.10 and Figure 5.11.

Step 1. Eliciting nonlinear probability weighting function.

By means of GA simulation (with an appropriate stopping rule before convergence), varying gamma and p, find the allotting pattern that minimizes f(w(p))= L - U, where w(p) is a probability weighting function, and p is a probability distribution over the five alternatives. Given gamma and p, I will explain its substeps in detail.

1-1. Let x=(x[1],⋯,x[5]) denote a given pattern of allotments, where 0≦ x[k] ≦10, k = 1,⋯,5, Σ x[k] = 10.

1-2. Let gamma be the parameter of weighting function, w(p) = w(p;gamma) = Exp(-(-Ln p)^gamma), 0<gamma≦1.

1-3. Define the extended utility function v(y;w(p),x,k) = rdeu(y;w(p)) - k * M * distance(y, x), k= 0,⋯,40, where M is a small constant fixed.

1-4. For each k, using linear optimization tool (such as SOLVER), compute a maximal pattern y*[k]= arg_max(v(y;w(p),x,k)).

1-5. Define the two functions L, U as follows. L is the latest index of y*[k], and so it is of the largest penalty, to keep the decision maker stuck to the pattern y*[0], a perfectly decisive position which optimizes v(y;w(p),x,0)= rdeu(y;w(p)). And U is the first index of y*[k], and so it is of the smallest size of penalty, to keep the optimality of x, the pattern observed, with respect to v.

1-6. Define the target function f(w(p))= L - U, as the measure of decisiveness (or the degree of precommitment) of the decision maker.

1-7. Output the argmax of the above target function, and stop.

Step 2. Simulate the curvature by approximated consonant beliefs.

Find a constant in unit interval, alpha, and a neally consonant 2-monotone convex function, v, such that you can manage to fit both the composition, v” = alpha * v + (1 - alpha) * v’ where v ’ is its conjugate function, and the basic probability mass corresponds to v by means of M vius inversion, with the curvature of probability weighting function and probability which have found in step 1, respectively.

Figure 5.5 Algorithm of the elicitation-approximation procedure

#	A_1	A_2	A_3	A_4	A_5	Event and it's Meaning	Count	v (A)	m(A)	v*(A)←	v' (A)	v (⊢A)	v' (⊢A)
0	0	0	0	0	0	never get right	1	7.8%	0.0%	0.0%	0.7%	99.3%	92.2%
1	1	0	0	0	0	get right by 1st pJudge	2	18.4%	18.4%	95.1%	95.8%	4.2%	81.6%
2	1	1	0	0	0	get right by 2nd pJudge	1	47.0%	28.5%	96.8%	97.5%	2.5%	53.0%
3	1	1	1	0	0	get right by 3rd pJudge	1	64.8%	17.9%	97.8%	98.6%	1.4%	35.2%
4	1	1	1	1	0	get right by 4th pJudge	2	77.5%	12.7%	98.5%	99.3%	0.7%	22.5%
5	1	1	1	1	1	get right by 5th pJudge	2	99.3%	13.6%	99.3%	92.2%	7.8%	0.7%
6	1	1	1	1	1	* not yet begin	2	100.0%	0.0%	99.3%	92.2%	7.8%	0.0%
7	1	0	0	0	0	* 1 is the right answer	2	0.0%	0.0%	95.1%	95.8%	4.2%	100.0%
8	0	1	0	0	0	2 is the right answer	1	0.0%	0.0%	76.1%	76.8%	23.2%	100.0%
9	0	0	1	0	0	3 is the right answer	1	0.0%	0.0%	47.7%	48.5%	51.5%	100.0%
10	0	0	0	1	0	4 is the right answer	1	0.0%	0.0%	30.0%	30.8%	69.2%	100.0%
11	0	0	0	0	1	5 is the right answer	2	0.7%	0.7%	21.7%	22.5%	77.5%	99.3%
12	0	1	1	1	1	1 is not right	2	4.2%	0.4%	80.8%	81.6%	18.4%	95.8%
13	0	0	1	1	1	not found by 2nd pJudge	1	2.5%	0.5%	52.3%	53.0%	47.0%	97.5%
14	0	0	0	1	1	not found by 3rd pJudge	1	1.4%	0.7%	34.4%	35.2%	64.8%	98.6%
15	0	0	0	0	1	* not found by 4th pJudge	2	2.3%	0.0%	21.7%	22.5%	77.5%	97.7%
16	0	1	1	1	1	* 1 is not the right answer	2	1.9%	0.0%	80.8%	81.6%	18.4%	98.1%
17	1	0	1	1	1	2 is not the right answer	1	23.2%	0.6%	99.3%	100.0%	0.0%	76.8%
18	1	1	0	1	1	3 is not the right answer	1	51.5%	0.6%	99.3%	100.0%	0.0%	48.5%
19	1	1	1	0	1	4 is not the right answer	1	69.2%	0.6%	99.3%	100.0%	0.0%	30.8%
20	1	1	1	1	0	* 5 is not the right answer	2	0.4%	0.0%	98.5%	99.3%	0.7%	99.6%
21	1	0	1	0	0	21	1	18.4%	0.0%	97.0%	97.7%	2.3%	81.6%
22	1	0	0	1	0	22	1	18.4%	0.0%	97.2%	97.9%	2.1%	81.6%
23	1	0	0	0	1	23	1	19.7%	0.6%	99.3%	100.0%	0.0%	80.3%
24	1	1	0	1	0	24	1	47.0%	0.0%	98.0%	98.7%	1.3%	53.0%
25	1	1	0	0	1	25	1	49.3%	0.6%	99.3%	100.0%	0.0%	50.7%
26	1	0	1	1	0	26	1	18.4%	0.0%	98.1%	98.8%	1.2%	81.6%
27	1	0	1	0	1	27	1	20.8%	0.6%	99.3%	100.0%	0.0%	79.2%
28	1	0	0	1	1	28	1	21.0%	0.6%	99.3%	100.0%	0.0%	79.0%
29	0	1	1	0	0	29	1	0.0%	0.0%	78.3%	79.0%	21.0%	100.0%
30	0	1	0	1	0	30	1	0.0%	0.0%	78.4%	79.2%	20.8%	100.0%
31	0	1	0	0	1	31	1	1.2%	0.4%	80.8%	81.6%	18.4%	98.8%
32	0	1	1	0	1	32	1	2.1%	0.4%	80.8%	81.6%	18.4%	97.9%
33	0	1	1	1	0	33	1	0.0%	0.0%	79.5%	80.3%	19.7%	100.0%
34	0	0	1	1	0	34	1	0.0%	0.0%	50.0%	50.7%	49.3%	100.0%
35	0	0	1	0	1	35	1	1.3%	0.5%	52.3%	53.0%	47.0%	98.7%
36	0	1	0	1	1	36	1	2.3%	0.4%	80.8%	81.6%	18.4%	97.7%
	19	19	19	19	19			0.0%	**99.3%**				

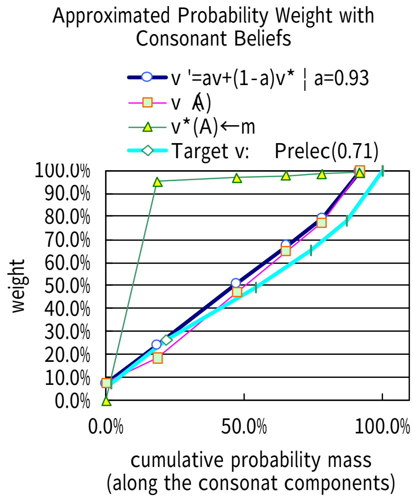
Figure 5.6 An example of (2-monotone convex) consonant belief approximation.

As shown in Figure 5.6, an inverse S shaped nonlinear weighting function with its marginal contributions has approximated (using Excel + Solver + Evolver) in accordance with the search order (not the ranking of outcomes). And the basic probability masses are computed by M bus inversion, which is the technique in game theory to compute agent ' s contribution against a given coalition. The convexity (/concavity) dispersions from additive probability measure of the consonant approximation for that weight (that is the square sum of the sum of dispersions in each column and row) are positive (/nearly 0).

自己配点テストにおける曖昧信念とその更新（2001/8/18← **Approximating Probability Weights** param **Total_error** 39.6 a= 0.9263

20 Aug 基本部分。初期容量の単調性チェックと反転公式の **v '=av+(1-a)v* ¦ a=0.93** 2 **0.71** (↓^10^4) **v '=av+(1-a)v***

ID #	A_1	A_2	A_3	A_4	A_5	m_cumulative	Target m	m_Error	Target v: Prelec(0.71)	v_Error	Predict m(A)	Predict v (A)	v'	m '(A)
	choice at the k-th pJudge							27.9808		11.5750				
0	0	0	0	0	0	2.0%	2.0%	-2.0%	0.0718	0.0%	0.0%	7.8%	7.2%	0.0%
1	1	0	0	0	0	22.0%	20.0%	-1.6%	0.2612	-2.0%	18.4%	18.4%	24.1%	24.1%
2	1	1	0	0	0	54.0%	32.0%	-3.5%	0.4921	1.4%	28.5%	47.0%	50.6%	21.0%
3	1	1	1	0	0	74.0%	20.0%	-2.1%	0.6528	2.0%	17.9%	64.8%	67.3%	19.8%
4	1	1	1	1	0	87.0%	13.0%	-0.3%	0.7814	0.9%	12.7%	77.5%	79.1%	9.8%
5	1	1	1	1	1	100.0%	13.0%	0.6%	1.0000	-0.7%	13.6%	99.3%	99.3%	13.6%
6	1	1	1	1	1	100.0%	0.0%	0.0%	1.0000	-0.1%	0.0%	100.0%	99.9%	0.0%
7	1	0	0	0	0		100.0%		Exp(-(-Ln p)^a)		0.0%	0.0%	7.0%	0.0%
8	0	1	0	0	0						0.0%	0.0%	5.6%	5.6%
9	0	0	1	0	0						0.0%	0.0%	3.5%	3.5%
10	0	0	0	1	0						0.0%	0.0%	2.2%	2.2%
11	0	0	0	0	1						0.7%	0.7%	2.3%	2.3%
12	0	1	1	1	1						0.4%	4.2%	9.8%	-0.6%
13	0	0	1	1	1						0.5%	2.5%	6.1%	1.6%
14	0	0	0	1	1						0.7%	1.4%	3.8%	-0.7%
15	0	0	0	0	1						0.0%	2.3%	3.7%	0.0%
16	0	1	1	1	1						0.0%	1.9%	7.8%	0.0%
17	1	0	1	1	1						0.6%	23.2%	28.8%	-0.5%
18	1	1	0	1	1						0.6%	51.5%	55.0%	-0.5%
19	1	1	1	0	1						0.6%	69.2%	71.4%	-0.5%
20	1	1	1	1	0						0.0%	0.4%	7.7%	0.0%
21	1	0	1	0	0						0.0%	18.4%	24.2%	-3.4%
22	1	0	0	1	0						0.0%	18.4%	24.2%	-2.1%
23	1	0	0	0	1						0.6%	19.7%	25.6%	-0.8%
24	1	1	0	1	0						0.0%	47.0%	50.7%	2.0%
25	1	1	0	0	1						0.6%	49.3%	53.0%	1.7%
26	1	0	1	1	0						0.0%	18.4%	24.3%	2.0%
27	1	0	1	0	1						0.6%	20.8%	26.6%	1.7%
28	1	0	0	1	1						0.6%	21.0%	26.8%	1.7%
29	0	1	1	0	0						0.0%	0.0%	5.8%	-3.4%
30	0	1	0	1	0						0.0%	0.0%	5.8%	-2.0%
31	0	1	0	0	1						0.4%	1.2%	7.0%	-0.9%
32	0	1	1	0	1						0.4%	2.1%	7.9%	1.5%
33	0	1	1	1	0						0.0%	0.0%	5.9%	2.0%
34	0	0	1	1	0						0.0%	0.0%	3.7%	-2.0%
35	0	0	1	0	1						0.5%	1.3%	5.0%	-0.8%
36	0	1	0	1	1						0.4%	2.3%	8.0%	1.5%
	19	19	19	19	19						99.3%			99.3%



shift graph of v ' vertically 0

Figure 5.7 A consonant belief approximation of inverse S-shaped probability weights.

The approximated inverse S shaped weight by linearcombination of the two-monotone convex capacity found in Figure 5.6 and its conjugate concave function with pessimistic ratio in the interval [0, 1]. The total error to be minimized is the sum of “m-error” and “v-error”. The “m-error” (and “v-error”) is the square sum of discrepancies from the basic probability assignment (the probability weighting function) that has elicited in the first step, in this case, as of a Prelec’s function.

As we can observe in Figure 5.7, the approximated weight rather fitted to the target weights at least vertically, and so the small v-error. And also this can be obtained at a sacrifice of the exactness from the consonant beliefs, which is graphically represented as the horizontal diecrepancies, so the m-error.

Therefore, I will consider this off-the-consonant-belief masses represent a model of partial knowledge of decision maker, who would be decomposed into the distributed micro knowledges “on the spots”. These micro knowledges are of “on the spot” because they become accessible dynamically only when each of which has activated by the arrival of local information corresponds to that event. But among them, conflict may ocuur.

	Elicitation for the nonlinear probability weights of RDEU/CPT-maximizer for experimental data of the "jiko-haiten" test ; 20(									
	alternatives	**A_1**	**A_2**	**A_3**	**A_4**	**A_5**	**A_0**	0誤差限界		
	search order (k-th pJudge)	1	2	3	4	5	NA	SUM	residual	
	probability mass	20%	20%	20%	20%	20%	0%	1	0	0
	cumulative	20%	40%	60%	80%	100%	100%	p_error*		0
	Prelec's prob. weighting (0.71)	25%	39%	54%	71%	100%	100%	**0.00**		
	marginal contribution to weigh	**25%**	14%	15%	17%	29%	0%			
	conditional probabilities	20%	25%	33%	50%	100%	0%	-		0-tolelance
	ranking of points	*6*	*2*	*3*	*3*	*1*	*3*	SUM	residual	1E-15
	your allotment (haiten)	**0**	**0**	**0**	**0**	**10**	0	10.0	0	
	inequality	*-2.0*	*-4.0*	*-6.0*	*-8.0*	*0.0*	*ieq=*	*-20*		penalty weight
	experimental data to fit	**2**	**4**	**2**	**1**	**1**	**0**	x_error		for x_error
		MAX	MIN	Range	EV	target parameter		**106.00**		0
		10	0	10	2	Prelec's fu	0.71		*result*	
	power utility (normalized)	0.00	0.00	0.00	0.00	1.00	0.00		upper	28
	expected_utility (power)	0.00	0.00	0.00	0.00	0.20	0.00	0.20	lower	41
	choquet integral (RDEU/CPT)	0.00	0.00	0.00	0.00	0.29	0.00	0.29	L-U	**13**
	parametors for simulation							-	--> min.	
@	weighting function	**2**	<== 1: convex/concave capacity, 2 or otherwise: Prelec function							
	convexity of power weightin	0.9962		*found by Evolver*		alpha	beta	gamma	gap_U	0.096
	optimistic coefficient (beta)	0.12			*(best value)*	0.7	0.7	0.7	upper	0.1960
	risk averse index (alpha)	1.00	compared models ↓		**EV**	**EU**	**Hurwicz**	**RDEU**	lower	0.2916
	Prelec's function (gamma)	**0.71**	*Penalty= 0.0*		3.20	0.41	0.70	0.38		
	target cell (maximization)	0.2916	***1***	EV	3.20	3.20	3.50	3.25	*simulation options*	
EV	**input your utility functional #**	**4**	RDEU	EU	0.41	0.41	0.35	0.41	number of intervals	
EU	1:EV, 2:EU, 3:Hurwicz, 4:RDEU, 5:Anxiety			Hurwicz	0.21	0.21	0.70	0.55		40
Hurwicz	optimize your allotment			RDEU	0.38	0.38	0.36	0.38	max penalty weight	
RDEU				(current)	2.00	0.20	0.12	0.29		0.02
Anxiety										

Figure 5.8 A snapshot of the spreadsheet simulation for eliciting nonlinear probability weights.

As the case in Figure 5.8, the best probability such that it minimizes the upper bound of the penalty weight to attain optimality of given pattern of allocation is even at the level of 20% among 5 options and thus any pattern of allocation optimizes the cumulative utility. Note that in this case the meaning of closedness of a pattern to the best, usually a decisive one, has transposed. It is a drawback of our elicitation method that any decisive (or indecisive) pattern of allocation as well as a flat allocation, can be rationalized by equal probabilities and near to linear weight over probabilities and by any nonlinear weight with even marginal contributions to cumulative probabilities. This deficiency may be handled, somewhat an ad hoc way, by enforcing some degree of preference for variation.

ID	weight	haiten1	haiter	haiter	haiter	haiter	haiter	target	EV	EU	Hurwicz	RDEU	Anxiety	model	penalty	p_error	x_error
1	0.02	2	4	2	1	1	0	0.1848	2	0.2	0.12698	0.187	0.20294	4	0.00216	4	0.10788
2	0.0195	2	4	2	1	1	0	0.1849	2	0.2	0.12678	0.1871	0.20291	4	0.00221	4	0.11348
3	0.019	2	4	2	1	1	0	0.1849	2	0.2	0.12656	0.1872	0.20289	4	0.00227	4	0.11953
4	0.0185	2	4	2	1	1	0	0.185	2	0.2	0.12633	0.1873	0.20286	4	0.00233	4	0.12608
5	0.018	2	4	2	1	1	0	0.1851	2	0.2	0.12609	0.1875	0.20283	4	0.0024	4	0.13318
6	0.0175	2	4	2	1	1	0	0.1851	2	0.2	0.12583	0.1876	0.2028	4	0.00247	4	0.1409
7	0.017	2	4	2	1	1	0	0.1852	2	0.2	0.12556	0.1878	0.20277	4	0.00254	4	0.14931
8	0.0165	2	4	2	1	1	0	0.1853	2	0.2	0.12527	0.1879	0.20273	4	0.00262	4	0.1585
9	0.016	2	4	2	1	1	0	0.1854	2	0.2	0.12497	0.1881	0.2027	4	0.0027	4	0.16856
10	0.0155	2	4	2	1	1	0	0.1855	2	0.2	0.12464	0.1882	0.20266	4	0.00278	4	0.17961
11	0.015	2	4	2	1	1	0	0.1856	2	0.2	0.1243	0.1884	0.20262	4	0.00288	4	0.19178
12	0.0145	2	4	2	1	1	0	0.1857	2	0.2	0.12393	0.1886	0.20257	4	0.00298	4	0.20523
13	0.014	2	4	2	1	1	0	0.1858	2	0.2	0.12353	0.1888	0.20252	4	0.00308	4	0.22015
14	0.0135	2	4	2	1	1	0	0.1859	2	0.2	0.1231	0.1891	0.20247	4	0.0032	4	0.23676
15	0.013	2	4	2	1	1	0	0.186	2	0.2	0.12264	0.1893	0.20242	4	0.00332	4	0.25533
16	0.0125	2	4	2	1	1	0	0.1861	2	0.2	0.12215	0.1896	0.20236	4	0.00345	4	0.27616
17	0.012	2	4	2	1	1	0	0.1863	2	0.2	0.12161	0.1899	0.20229	4	0.0036	4	0.29966
18	0.0115	2	4	2	1	1	0	0.1864	2	0.2	0.12103	0.1902	0.20222	4	0.00375	4	0.32628
19	0.011	2	4	2	1	1	0	0.1866	2	0.2	0.12039	0.1905	0.20215	4	0.00392	4	0.35661
20	0.0105	2	4	2	1	1	0	0.1868	2	0.2	0.11969	0.1909	0.20206	4	0.00411	4	0.39139
21	0.01	2	4	2	1	1	0	0.187	2	0.2	0.11892	0.1913	0.20197	4	0.00432	4	0.4315
22	0.0095	2	4	2	1	1	0	0.1872	2	0.2	0.11808	0.1918	0.20187	4	0.00454	4	0.47812
23	0.009	2	4	2	1	2	0	0.1875	2	0.2	0.11713	0.1923	0.20176	4	0.00479	4	0.53272
24	0.0085	2	4	2	1	2	0	0.1878	2	0.2	0.11608	0.1928	0.20163	4	0.00508	4	0.59724
25	0.008	2	4	2	1	2	0	0.1881	2	0.2	0.11489	0.1935	0.20149	4	0.00539	4	0.67422
26	0.0075	2	4	2	1	2	0	0.1884	2	0.2	0.11355	0.1942	0.20133	4	0.00575	4	0.76712
27	0.007	2	4	2	1	2	0	0.1888	2	0.2	0.11202	0.195	0.20114	4	0.00616	4	0.88062
28	0.0065	2	4	2	1	2	0	0.1893	2	0.2	0.11025	0.196	0.20093	4	0.00664	4	1.02131
29	0.006	2	4	2	1	2	0	0.1899	2	0.2	0.10818	0.1971	0.20068	4	0.00719	4	1.19862
30	0.0055	2	3	2	1	2	0	0.1905	2	0.2	0.10574	0.1984	0.20039	4	0.00785	4	1.42646
31	0.005	2	3	1	1	2	0	0.1913	2	0.2	0.10281	0.1999	0.20004	4	0.00863	4	1.72601
32	0.0045	3	3	1	1	2	0	0.1923	2	0.2	0.09922	0.2019	0.19961	4	0.00959	4	2.13088
33	0.004	3	3	1	1	2	0	0.1935	2	0.2	0.09475	0.2043	0.19907	4	0.01079	4	2.6969
34	0.0035	3	3	1	1	2	0	0.195	2	0.2	0.08899	0.2073	0.19838	4	0.01233	4	3.52248
35	0.003	3	3	1	1	3	0	0.1971	2	0.2	0.08131	0.2114	0.19747	4	0.01438	4	4.79448
36	0.0025	3	3	1	0	3	0	0.1999	2	0.2	0.07093	0.2172	0.19618	4	0.01726	4	6.90405
37	0.002	3	3	1	0	3	0	0.2043	2	0.2	0.06234	0.2258	0.19425	4	0.02158	4	10.7876
38	0.0015	4	2	0	0	4	0	0.2114	2	0.2	0.04981	0.2402	0.19103	4	0.02877	4	19.1767
39	0.001	4	1	0	0	5	0	0.2248	2	0.2	0.06084	0.2611	0.18409	4	0.03632	4	36.3234
40	0.0005	3	0	0	0	7	0	0.249	2	0.2	0.07913	0.2769	0.17673	4	0.02792	4	55.8447
41	0	0	0	0	0	10	0	0.2916	2	0.2	0.11681	0.2916	0.16565	4	0	4	106

Figure 5.9 A counterfactual ranking and decisiveness measure.

Figure 5.9 shows a simulated counterfactual ranking of the optimal allocation patterns using spreadsheet model in Figure 5.8 and SOLVER which is the standard optimization tool of Excel (a product of Microsoft), iteratively executed by VBA macro I coded. Given allocation vector, eg. 2-4-2-1-1-0 (case 14, Q4), decisiveness measures are computed for each candidate inverse S shape probability weighting function with a fixed index (0.71 in Figure 5.8). During the simulation, the same target cell that represents REDU-like cumulative utility (but ranked by search order, not by outcomes) added by the penalty term for "x_error" (i.e., the square sum of approximation error) with varying linear coefficient to be maximized. Exploiting this counterfactual ranking of optimal allocation patterns, we get decisiveness measure for probability weighting function. In Figure 5.9, the lower bound of linear weight of penalty where the given pattern, 2-4-2-1-1-0, is still optimal is 28, and the decisive allocation pattern with no penalty is 0-0-0-0-10-0 occured in the last row, 41. Therefore the decisiveness measure is 13 = 41-28 with tolerance, 1.0, in its x_error. (Allocations are rounded.)

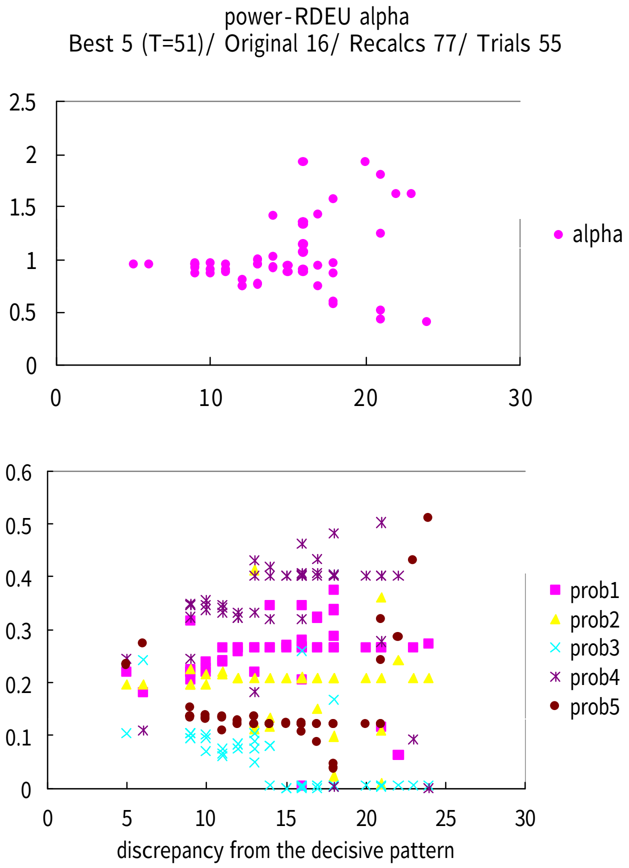


Figure 5.10 A result of elicitation by GA simulation --- a case of power-RDEU

In figure 5.10, the horizontal axis represents the minimum value of the penalty weight in order to justify the allocation pattern given probability weight. The tool for optimization I used in this simulation is, EVOLVER, the Palisade ' s software addin to Microsoft ' s EXCEL. And I used the default setting as for the genetic parameters, population size =20, cross over rate =0.5, mutation rate =0.06. The objective function and the constraints for the above elicitation procedure are depicted in the later figure 5.12. Evolver iteratively execute the VBA macro as stated in Figure 5.9 varying probabilities and (the parameter of) probability weighting functions so that it minimizes the decisiveness measure over the RDEU models with respect to the monotone capacities.

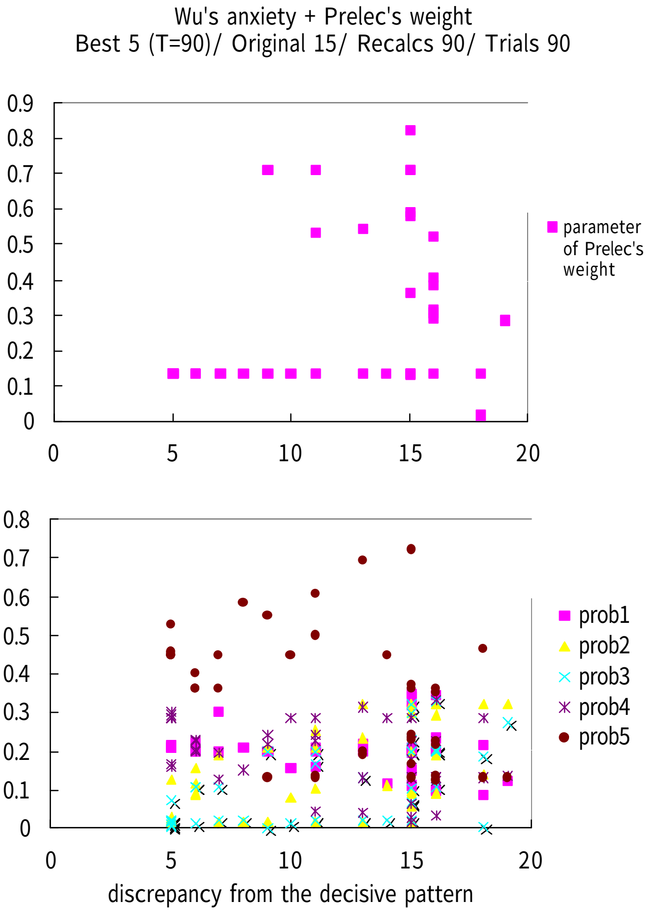


Figure 5.11 Another result of elicitation by GA simulation --- a case of the anxiety model, with RDEU (CPT) assuming Prelec's weighting function, Exp(-(-Ln p)^a).

In Figures 5.10-11, sample outputs of the elicitation for the nonlinear probability weights by means of GA simulation have shown.

6. Conclusion

In this paper, RDEU / CPT models with inverse S shaped probability weight are informally interpreted into the decentralized search process of decision maker with uncertain knowledge base (or evidential corpus). Based on these cumulative representations, some preliminary experimentations using human subjects and computer simulation models of elicitation of distributed knowledge of decision makers in the iterative multi-choice test has demonstrated. I found the notion of decisiveness that it is based on counterfactual ranking of weighting functions, and consonant belief approximation for nonlinear probability weighting are both useful, but it seems that further investigation is needed.

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